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RF-3615

M. Sc. (Part - II) Examination

April / May – 2010

Advance Integral Transforms - 5006

Time : Hours] [Total Marks :

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. SC. - 2**

Name of the Subject :
☛ **ADVANCE INTEGRAL TRANSFORMS - 5006**

☛ Subject Code No. : 3 6 1 5 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Notations used are standard.
(4) Figure at the right end of the first line of each question indicate full marks.

Q.1	**(a)**	State and prove inverse theorem for Mellin transform.	[6]
	(b)	Define Inverse Mellin transform. State and prove convolution theorem for Mellin transform.	[4]
	(c)	Prove that $M[f^n(x):s] = \frac{(-1)^n(s-1)!}{[(s-n)-1]!} f^*(s-n)$.	[4]
		OR	
Q.1	**(a)**	Find the potential $\emptyset(r,\theta)$ that satisfies the linear equation $r^2\emptyset_{rr} + r\emptyset_r + \emptyset_{\theta\theta} = 0$ in an infinite wedge $0 < r < \infty;\ -\alpha < \theta < \alpha$	[6]
	(b)	with boundary conditions $\emptyset(r,\alpha) = f(r);\ \emptyset(r,-\alpha) = g(r); 0 \le r < \infty$, $\emptyset(r,\theta) \to 0$ as $r \to \infty\ \forall\ \theta \in (-\alpha,\alpha)$.	[4]
	(c)	In usual notation prove that $M\left[\frac{\Gamma a}{(1+x)^a} : s\right] = \Gamma(a-s)\,\Gamma s$ and deduce $M\left[\frac{1}{(1+x)} : s\right] = \pi \operatorname{cosec}(\pi s)$.	[4]
Q.2	**(a)**	Let $\bar{f}_n(k)$ and $\bar{f}'_n(k)$ be the Hankel transform of order n of functions f(r) and $f'(r)$ respectively then $\bar{f}'_n(k) = -\frac{k}{2n}[(n+1)\bar{f}_{n-1}(k) - (n-1)\bar{f}_{n+1}(k)]$.	[6]
	(b)	Find Hankel transform of order n of $\frac{d^2f}{dr^2} + \frac{1}{r}\frac{df}{dr} - \frac{n^2}{r^2}f$.	[4]
	(c)	State and prove Parseval's relation for Hankel transform.	[4]

OR

Q.2	(a)	The magnetic potential υ of the circular disc of radius 'a' and strength 'ω' magnetized parallel to axis satisfies Laplace equation and it is equal to $2\pi\omega$ on the disc itself and it vanishes at the exterior points in the plane of a disc. Show that at point (r,z) and z > 0 $\upsilon = 2\pi\omega a \int_0^{\infty} e^{-kz} J_0(kr) J_1(ak) dk$.	[6]
	(b)	Find Hankel transform of $f(r) = \begin{cases} a^2 - r^2; 0 < r \le a \\ 0; \ r \ge a \end{cases}$; $n = 0$.	[4]
	(c)	State and prove Change of scale property for Hankel transform.	[4]
Q.3	(a)	Use Finite Hankel transform to solve differential equation $\frac{\partial^2 \upsilon}{\partial r^2} + \frac{1}{r}\frac{\partial \upsilon}{\partial r} = \frac{1}{k_1}\frac{\partial \upsilon}{\partial t}; 0 \le r \le 1; t > 0$ where $\upsilon = \upsilon_0 =$ constant for r =1; υ=0 for t = 0 and 0 ≤ r <1	[6]
	(b)	Show that Finite Hankel transform , $H_0^*(r^2) = \frac{a^3}{k} J_1(ka) - \frac{2a^2}{k^2} J_2(ka)$.	[4]
	(c)	Find the Finite Hankel transform of r^{n-1} where $r J_{n-1}(kr)$ is the kernel of transform (n >1).	[4]
		OR	
Q.3	(a)	Show that $\int_0^1 \frac{J_n(ar)}{J_n(a)} r J_n(kr) dr = \frac{k}{a^2-k^2} J'_n(k)$ where $J_n(k) = 0$.	[6]
	(b)	Prove that Finite Hankel transform, $H_n^*\left[\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr}\right] = \frac{k}{2}\left[H_{n+1}^*\left(\frac{df}{dr}\right) - H_{n-1}^*\left(\frac{df}{dr}\right)\right]$	[4]
	(c)	Find the Finite Hankel transform of order zero of a constant function.	[4]
Q.4	(a)	Solve the Initial value problem $u_{n+2} - 5u_{n+1} + 6u_n = 2^n$; $u_0 = 1$ and $u_1 = 0$ using Z-transform.	[6]
	(b)	If $Z[u_n] = \frac{2z^2+3z+4}{(z-1)^3}$. Find u_1 and u_2.	[4]
	(c)	Find (i) $Z^{-1}\left[\frac{3z^2-z}{(z-1)(z-2)^2}\right]$ and (ii) $Z^{-1}\left[\frac{z}{(z-1)(z-2)}\right]$..	[4]
		OR	
Q.4	(a)	Prove that $Z[n^p] = -z \frac{d}{dz}[Z(n^{p-1})]$ where p >0 is any integer and hence deduce $Z[n] = \frac{z}{(z-1)^2}$ and $Z[n^2] = \frac{z^2+z}{(z-1)^3}$.	[6]
	(b)	State and prove Shifting theorem for Z-transform.	[4]
	(c)	Find $Z[\cos(n\theta)]$.	[4]
Q.5	(a)	Define Hilbert Transform. Also derive the formula for Inverse Hilbert transform.	[6]
	(b)	Find the Hilbert transform of $f(t) = \cos(\omega t)$.	[4]
	(c)	If $f(t) = e^{-at}$ find Generalized Stieltjes transform of f (t).	[4]

OR

Q.5	(a)	Find the solution of the integral equation given by $\lambda \int_0^{\infty} \frac{f(t)}{t+x} dt = f(x)$ where λ is a real parameter, using Stieltjes transform.	[6]
	(b)	Find Hilbert transform of $f(t) = \frac{t}{t^2+a^2}; a > 0$.	[4]
	(c)	Find Stieltjes transform of (i) $f(t) = (t+a)^{-1}$ and (ii) $f(t) = t^{\alpha-1}$.	[4]